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RESEARCH ARTICLE

Statistical calibration via Gaussianization in hot-wire anemometry

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Abstract A statistical method is introduced, that is based on Gaussianization to estimate the nonlinear calibration curve of a hot-wire probe, relating the input flow velocity to the output (measured) voltage. The method uses as input a measured sequence of voltage samples, corresponding to different unknown flow velocities in the desired operational range, and only two measured voltages along with their known (calibrated) flow velocities. The method relies on the conditions that (1) the velocity signal is Gaussian distributed (or has another known distribution), and (2) the measured signal covers the desired velocity range over which the sensor is to be calibrated. The novel calibration method is validated against standard calibration methods using data acquired by hot-wire probes in wind-tunnel experiments. In these experiments, a hot-wire probe is placed at a certain region downstream of a cube-shaped body in a freestream of air flow, properly selected, so that the central limit theorem, when applied to the random velocity increments composing the instantaneous velocity in the wake, roughly holds, and renders the measured signal nearly Gaussian distributed. The statistical distribution of the velocity field in the wake is validated by mapping the first four statistical moments of the measured signals in different regions of the wake and comparing them with corresponding moments of the Gaussian distribution. The experimental data are used to evaluate the sensitivity of the method to the distribution of the measured signal, and the method is demonstrated to possess some robustness with respect to deviations from the Gaussian distribution.

1 Introduction

Hot-wire anemometry is used to measure the fluid local velocity with high temporal resolution and fine scales of velocity fluctuations. It is used extensively in experimental studies of fluids (Stainback and Nagabushana 1993). The output of the hot-wire probe is analogue voltage that is related through a nonlinear mapping to the flow velocity. Thus, before using the hot-wire probe for measuring fluid velocity, it should be calibrated. Commonly, this calibration is done using an accurate Pitot probe. The measured velocity V obtained by the Pitot probe is related to the output voltage E as measured by the hot-wire probe at (theoretically) identical flow conditions (temperature and location). The calibration curve is fitted to the data points and used as a mapping function g, that transforms the voltage signal, measured by the probe, to velocity signal, i.e., $V = g(E)$ (e.g., Jørgensen 2012).

There are several techniques to obtain the fitting function g. The first is polynomial curve fitting, where, usually, a polynomial of fourth order is used to fit the data points. In this case, the fitting error is less than 1%. For polynomial fitting of nth order, we need at least $n + 1$ points. Inappropriate spacing between the data points or excessive-order fitting may generate a fitting curve that oscillates between the data points. The second method is a power law fitting:

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$$E^2 = A + BV^n, \quad (1)$$

where A and B are constants that depend on the fluid and sensor physical properties and operating temperatures, and n is a constant that depends on the sensor dimensions and satisfies $0.45 \leq n \leq 0.52$. This fitting function is based on King's Law (King 1914) that provides a relationship between power, dissipated in a hot-wire anemometer through convection, and the resulting temperature difference between the wire and the ambient fluid.

In both techniques, to obtain the fitting function g, N data points $\{(E_i, V_i)\}_{i=1}^N$ should be acquired throughout the desired velocity range, where, for each data point, E_i is the output voltage corresponding to the introduced velocity V_i. These N data points are then plotted on the V vs. E plane. For proper calibration of the hot-wire probe, it is common to use $N \geq 6$ data points, although $N \geq 10$ data points are recommended in Jørgensen (1996), where N is chosen according to the velocity range for which the probe is used for.

In long-duration experiments, the nonlinear mapping of the hot-wire sensor may vary due to changes in the experimental conditions and sensor drifts, thus requiring recalibration during the experiments. This recalibration can be done outside the test section, but, as is well-known, it is always preferable (if possible) to calibrate the hot-wire probe within the same test section of the wind tunnel where it would eventually be used, thus saving valuable time and introducing less interference into the experimental setup. On the other hand, recalibration in situ can be a time-consuming process, even with modern, computer-controlled wind-tunnel experiments, because it requires adjusting the wind-tunnel velocity to obtain all required calibration data points. These issues render the calibration process costly in terms of time and resources, and motivate the search for a fast and less demanding calibration method.

Several fast methods have been proposed for recalibration of a sensor that was initially properly calibrated prior to conducting the experiments, via applying a correction to the initial calibration. Hultmark and Smits (2010) have proposed a recalibration method designed to handle temperature drift during the experiment. Recently, Talluru et al (2014) have demonstrated an on-the-fly recalibration method which is not restricted to only correct temperature drift. Based on a single recalibration point, this method is used to correct sensor drift in boundary layer measurements by placing the probe in the freestream in between experimental samples. Relying upon the existence of an accurate initial calibration, these methods are sensitive to severe sensor degradation, which might render the initial calibration irrelevant. Thus, to avoid calibration errors, complete calibration (as opposed to recalibration via correction) of the hot-wire during long experiments is preferable.

In this paper, we introduce a novel fast statistical calibration method that requires only two calibration data points, i.e., $N = 2$. Being fast and relatively undemanding, the method proposed herein provides a complete calibration of the hot-wire sensor without relying on prior calibration, and, therefore, it can be repeated, in situ, as often as required, e.g., in cases where multiple calibrations should be performed due to varying conditions and long durations.

The method relies on the statistical properties of turbulent flow. Accordingly, we consider a statistical method for recovering the nonlinear relation between the input velocity and the output voltage of a hot-wire sensor located inside a turbulent flow regime under certain conditions. In turbulence, one may consider the instantaneous velocity $V(t)$, measured by the hot-wire probe, as the sum of random velocity increments carried by fluid elements arriving from random displacements of small eddies. These incremental changes can be considered to be independent and identically distributed random variables (Tennekes and Lumley 1972). Consequently, by the well-known central limit theorem (CLT) (Trotter 1959), and assuming some technical conditions, the random variable $V(t)$ can be assumed to be nearly Gaussian distributed for all t based on the classical eddy model of turbulence (Lumley 1972). It should be noted that perfect Gaussian distributions should not, in general, be expected in homogeneous turbulent flows (Jimenez 1998). In the case of hot-wire anemometry, the nonlinear mapping f transforms the distribution of V. Consequently, the transformed random variable $E = f(V)$ is "less Gaussian" than the original function V. The following proposed method of calibration is based on this insight.

2 Calibration method

2.1 Mathematical foundation

The method of Gaussianization is based on the following well-known property of the cumulative distribution function (cdf) of any random variable.

Consider the random variable X, having F_X as its cdf. Then, the random variable $U \triangleq F_X(X)$ is uniformly distributed on $[0, 1]$ (Rohatgi 1976).

In particular, the standard (zero-mean, unit variance) Gaussian cdf (the Laplace function) $\Phi(u)$ transforms the standard Gaussian variable to the uniform random variable $U([0, 1])$. Therefore, $\Phi^{-1}(U([0, 1]))$ is the standard Gaussian random variable.

Consider the random variable:

$$Z \triangleq \Phi^{-1}(F_E(E)). \quad (2)$$

Obviously, Z is a standard Gaussian random variable. Because Z is obtained from V through $E = f(V)$, and V is

Gaussian, it is clear that both Gaussian random variables are linearly related, that is

$$Z = aV + b, \tag{3}$$

for some constants a and b. These constants can be determined using two calibration data points.

It should be noted that the method can be easily generalized for distributions other than the Gaussian one. Consider a random variable V, with a given cdf F_V. We can find Z from E by preforming,

$$Z = F_V^{-1}(F_E(E)). \tag{4}$$

Notice, however, that the usage of Eq. (4) relies on knowing F_V, which, typically, requires some physical insights into the problem on hand.

2.2 Calibration procedure

We now present the method to obtain the nonlinear mapping of the velocity signal recorded by a hot-wire probe. Our goal is to do it with only two known (calibrated) velocities, V_1 and V_2, corresponding to voltage outputs E_1 and E_2, respectively, in the recorded turbulent signal range.

The method consists of the following steps:

1. Provide two calibration data points $(E, V)_1$ and $(E, V)_2$ within the measured voltage range $[E_{\min}, \; E_{\max}]$ of the recorded hot-wire signal that is assumed to be Gaussian. That is, $E_{\min} < E_1 < E_2 < E_{\max}$.
2. Find the signal $Z(t)$ from signal $E(t)$ using Eq. (2), where $F_E(E(t))$ is computed from $E(t) \in [E_{\min}, \; E_{\max}]$.
3. Use the two known (calibration) data points from step 1 to evaluate the coefficients a and b in Eq. (3).
4. Retrieve the estimated signal $V(t)$ from $Z(t)$ using Eq. (3). Denote the retrieved value of V by $\widehat{V}$.

After preforming the above steps, one obtains $Z(t)$ and the estimated velocity $\widehat{V}(t)$ corresponding to the measured voltage $E(t)$. To estimate the nonlinear mapping function g, we consider first a 4th-order polynomial fitting of nine voltage samples E_i (equally spaced sample values extracted from the signal E in the range $E_{\min} < E_i < E_{\max}$) to their corresponding Z_i values. This results in five polynomial coefficients $p_{Z,k}$ $(k = 0, 1, \ldots, 4)$, that is

$$Z_i = \sum_{k=0}^{4} p_{Z,k} E_i^k, \quad i = 1, 2, \ldots, 9. \tag{5}$$

Second, we use the obtained coefficients a and b from Eq. (3) together with polynomial coefficients $p_{Z,k}$ to obtain the polynomial coefficients $p_{\widehat{g},k}$ that represent the function $\widehat{g}$, which is the estimate of the function g, that is

$$\widehat{V}_i = \widehat{g}(E_i) = \sum_{k=0}^{4} p_{\widehat{g},k} E_i^k, \quad i = 1, 2, \ldots, 9, \tag{6}$$

where

$$p_{\widehat{g},0} = \frac{1}{a}(p_{Z,0} - b) \tag{7}$$

and

$$p_{\widehat{g},k} = \frac{1}{a} p_{Z,k}, \quad k = 1, 2, 3, 4. \tag{8}$$

2.3 Main features of the method

The proposed method has several distinct advantages, which we discuss below. First, it can be used when the calibration data points are provided in a limited narrow velocity range, whereas the measurements are acquired in a wider velocity range that extends beyond the range spanned by the provided calibration data. Such cases occur, e.g., when a Pitot probe can only measure a limited velocity range, or when the required range of measured velocities is below the operational range of the wind tunnel, as is the case with measurements in boundary layers, or wakes. In such cases, our method can be used to extend the calibration curve beyond the provided calibration points, assuming that a velocity signal of a known distribution can be provided that covers the extended range.

Second, the new method takes advantage of present day's virtually unlimited data acquisition rates by relying on the statistical convergence of measured data. In experiments where the statistical properties of the turbulent flow are known, one can use the method after the turbulent velocity field has been acquired, or in between experimental samples as a recalibration method, without running separate experiments to refine or correct an existing calibration. The two calibration data points can then be obtained in the freestream flow, during or after the experiments themselves.

Third, in certain cases, the measured data are used to only obtain normalized differential values of measured velocities, e.g., when measuring the normalized standard deviation field of the flow. In such cases, knowing the absolute velocity field is not required, which obviates the need for providing the two calibration points, and the measured experimental signals are sufficient to estimate the probe's nonlinear mapping. Avoiding the need to obtain the two calibration data points leads, obviously, to considerable time and resource savings.

Finally, the new calibration method can also be used for studying new flow-fields where the distribution of the velocity signal is not known a priori. In such cases, an auxiliary setup for generating velocity signals having known distributions is required. The auxiliary setup should be mounted inside the wind-tunnel test section, such that it will not interfere with the studied flow during the experiments, or, in case it is impossible to eliminate such interference, the setup should be designed to be easily removable so that it can only be used in between long-duration experimental segments. As an example, a certain bluff body with known and mapped turbulent wake properties can be positioned at a certain location in the test section that renders it both accessible to the probe and non-interfering with the studied flow. After acquiring the two calibration data points with the aid of the bluff body, the hot-wire is traversed back to the investigated region of the new flow-field. This procedure, that can be automated and computer-controlled, should be faster than a conventional calibration procedure that requires varying the wind-tunnel velocity multiple times for obtaining sufficiently many calibration data points.

3 Validation

To validate the method, we compare it with standard calibration methods using hot-wire data acquired in the Technion's wind-tunnel laboratory. An example of a calibrated fit is presented in Fig. 1. Hot-Wire Anemometry system of A. A. Lab Systems is used. The probe itself is a tungsten wire of 5 μm diameter and 1.25 mm length (probe model: DANTEC, type 55P11, straight prongs). The wire is calibrated in the freestream of the wind tunnel using a Pitot-static tube in a velocity range up to 10 m/s, at a temperature of 20 °C. To these data points we fit and plot a 4th-degree polynomial to obtain the calibration curve $V = g(E)$.

To generate the desired Gaussian velocity signal $V(t)$, we place the probe downstream of a cube-shaped body in a freestream of air flow. The idea is to place the probe in a certain region of the generated wake downstream of the body, where we can achieve the desired signal properties for testing our calibration method. The specific properties we are looking for are:

1. The statistical distribution of the signal should be as close to Gaussian as possible.
2. The signal should cover the desired velocity range that should be calibrated, i.e., it should possess a sufficiently large standard deviation.

The experimental setup with the corresponding coordinate system is shown in Fig. 2. The cube is made of perspex with edge equal to $a_c = 20$ mm. The cube is placed inside the test section of the wind tunnel at a fixed location, mounted on a fixed rod connected to one of the cube side edges. The rod is placed on the side wall to enable placing the hot-wire probe behind the cube at a certain downstream location of its wake. The rod length and thickness are selected to be as thin as possible (approximately 1.65 mm thick, 40 mm long) to reduce its effect on the cube wake, but rigid enough to avoid oscillations of the cube due to freestream flow velocities up to 15 m/s. We choose to demonstrate our method on a 20 mm cube-shaped body, because it is easy to manufacture, and its associated Reynolds number is sufficiently high, so that it generates highly fluctuating signals.

Before testing the proposed method of calibration, we study the statistical properties of the generated wake. Three-dimensional (3D) mappings of the spatial structure of the wake flow-field of the cube-shaped body are

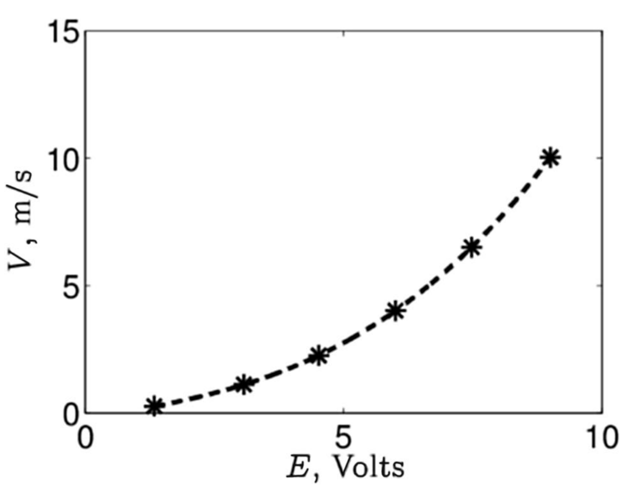

Fig. 1 Calibration data points (*asterisks*) and calibration curve (*dashed line*) obtained using 4th-degree polynomial fitting

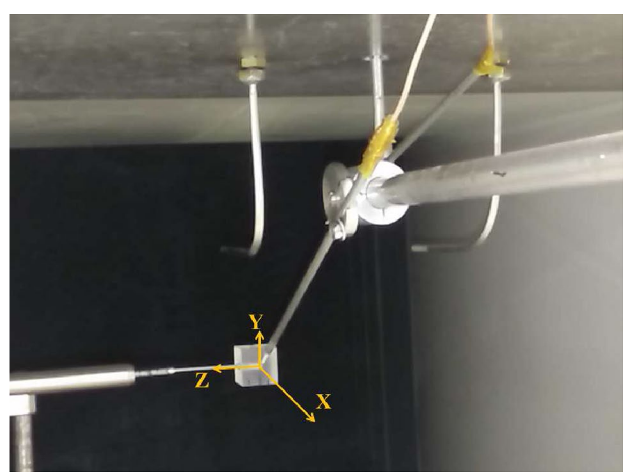

Fig. 2 Experimental setup: cubical body placed in a wind-tunnel test section. The origin of the associated coordinate system is at the center of the downstream face of the cube

obtained by hot-wire measurements of the streamwise velocity component. For this purpose, the probe is mounted on a three-axis traversing mechanism, having a resolution of 5 μm in the vertical direction, and 1 mm in the streamwise and spanwise directions. The positioning of the probe within the wake region at a given downstream station is done automatically using the computer to drive the stepping motors through the LabVIEW software. The flow is sampled at 1210 locations: 11 downstream locations in the range $x \in [5,\ 105]$ mm with 10 mm step size, 11 spanwise locations in the range $z \in [-25,\ 25]$ mm, and 10 vertical locations in the range $y \in [-20,\ 25]$ mm with 5 mm step size in both y and z directions to cover the wake. The 6 s velocity record is digitized at 1 kHz rate, giving 6×10^3 measurement points for each $(x,\ y,\ z)$ location. That a number of measurement points along with probe spatial resolution were found to be adequate for obtaining reliable high-order statistical moments and well-converged probability density functions (pdfs). The spatial resolution issues are discussed in detail in Hutchins et al. (2009). The probe wire length determines the attenuation levels of the smaller scale fluctuations of the flow, which plays a role in the acquired signal spectra appearance and its distribution. The probe used in our experiments is less than 1.25 mm long. According to Ligrani and Bradshaw (1987) this wire length is in the range that should be used for measuring kurtosis and skewness factors, as well as energy in wall turbulence measurements.

The wake measurement is conducted with three different freestream velocities: $U_\infty = 5, 7, 9$ m/s. For each

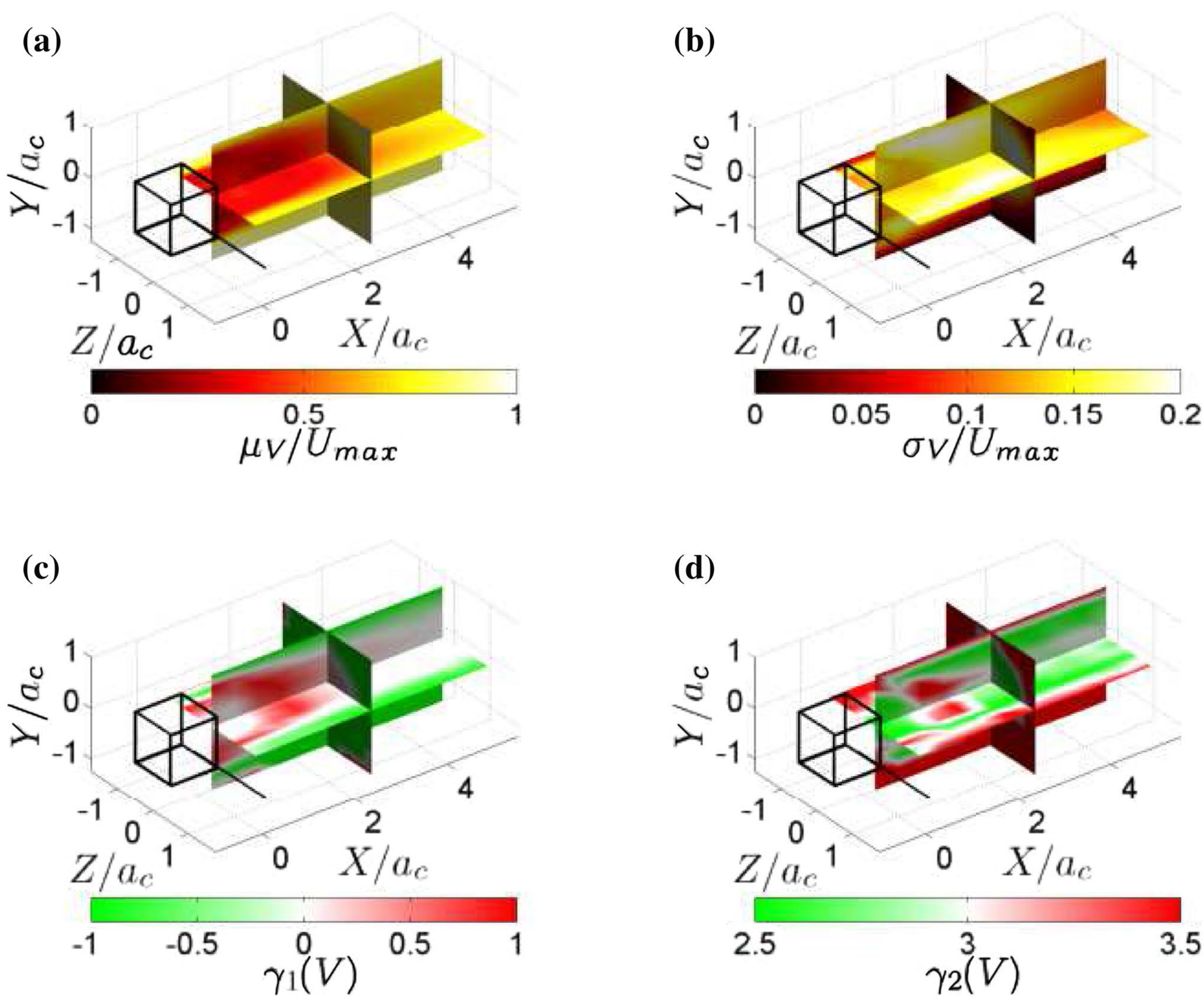

Fig. 3 Isometric view of x–y, x–z, and y–z slices of first four statistical moments of the wake of a cube with edge length of $a_c = 20$ mm, at freestream velocity $U_\infty = 5$ m/s: **a** mean velocity μ_V normalized by freestream velocity U_∞. **b** Standard deviation σ normalized by freestream velocity U_∞. **c** $\gamma_1(V)$. **d** $\gamma_2(V)$

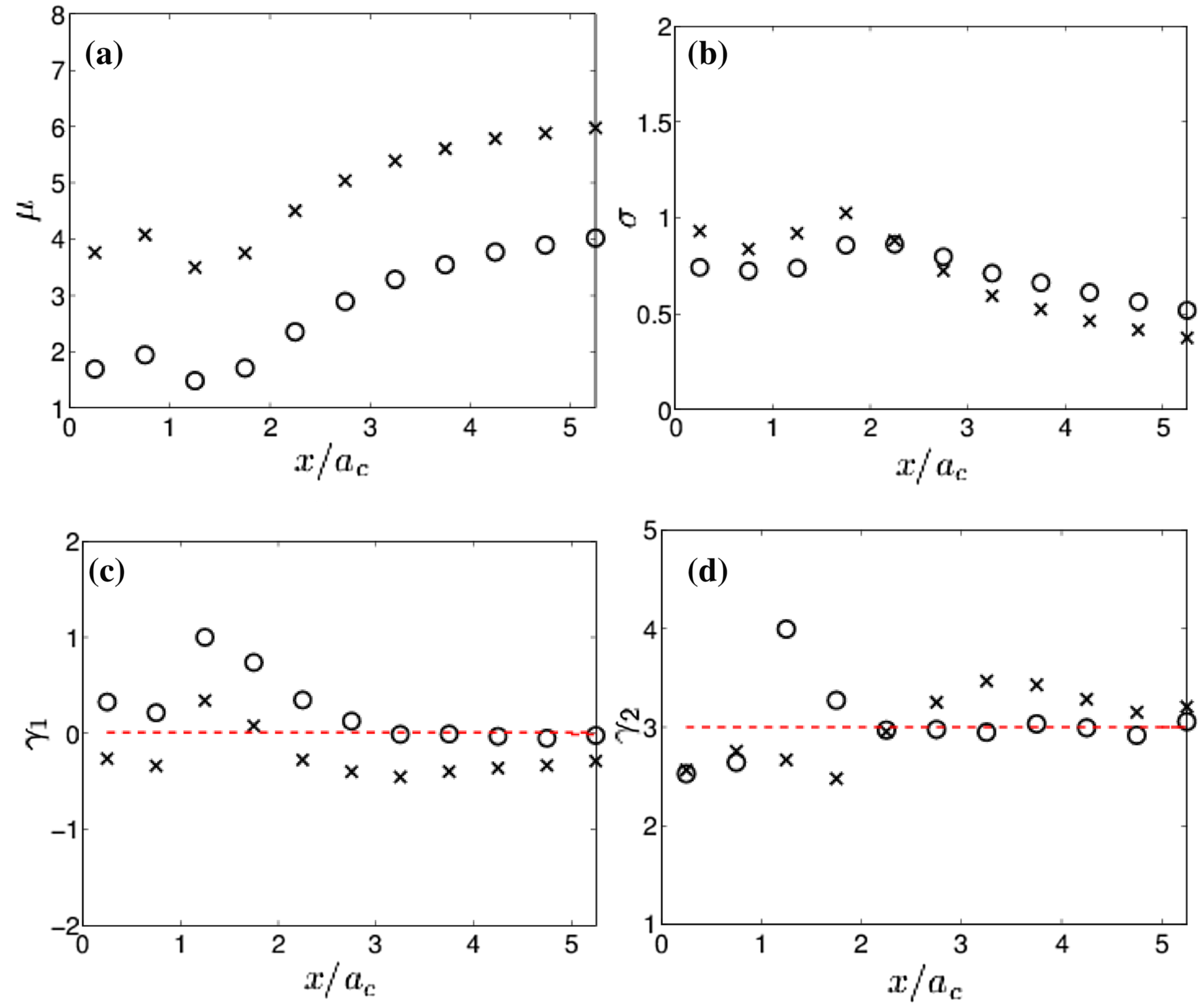

Fig. 4 Streamwise variation of the first four statistical moments of measured voltage E (*crosses*), and velocity V (*circles*) along the wake at $y = 0.25a_c$, $z = -0.25a_c$ and $U_\infty = 5$ m/s. $\gamma_1 = 0$ and $\gamma_2 = 3$ are denoted by *dashed* (*red*) *line*

experiment, we calibrate the probe by the traditional technique at the beginning and at the end of the experiment, to ensure that the hot-wire does not drift during the experiment. For each experiment, we compute the first four statistical moments of the acquired signals. These moments are: the mean $\mu_V = \mathbf{E}[V(t)]$, the standard deviation $\sigma_V = [\mathbf{E}(V - \mu_V)^2]^{0.5}$, the skewness $\gamma_1(V) = \mathbf{E}[(V - \mu_V)^3]/\sigma_V^3$, and the kurtosis, $\gamma_2(V) = \mathbf{E}[(V - \mu_V)^4]/\sigma_V^4$. A Gaussian signal should have skewness $\gamma_1 = 0$ and kurtosis $\gamma_2 = 3$ (Krishnan 2006).

In Fig. 3, we show the results obtained at a freestream velocity of $U_\infty = 5$ m/s, i.e., at Reynolds number $Re_a = 6.4 \times 10^3$, based on the cube edge length a_c and kinematic viscosity of $\nu = 1.57 \times 10^{-5}$ m^2/s. At these moderate Reynolds numbers, Fig. 3 shows that in the region just downstream the cube, the first statistical moment values, i.e., the mean velocity values, are lower than the freestream velocity. The regions of high shear can be identified by high σ_V values. The highest values occur approximately along the lines $z, y = \pm 0.25a_c$ and reach their maximum at about $x = 2a_c$. In the near field, the wake is highly unstable, resulting in skewed signals with kurtosis far from its Gaussian value. At downstream distances greater than $x = 3a_c$, the measured signal in the wake core has skewness and kurtosis close to the Gaussian values $\gamma_1 = 0$ and $\gamma_2 = 3$, i.e., the far wake core can be considered to consist of homogeneous turbulent flow. This region is suitable for applying our proposed calibration method. Similar results are obtained for freestream velocities $U_\infty = 7$ m/s and $U_\infty = 9$

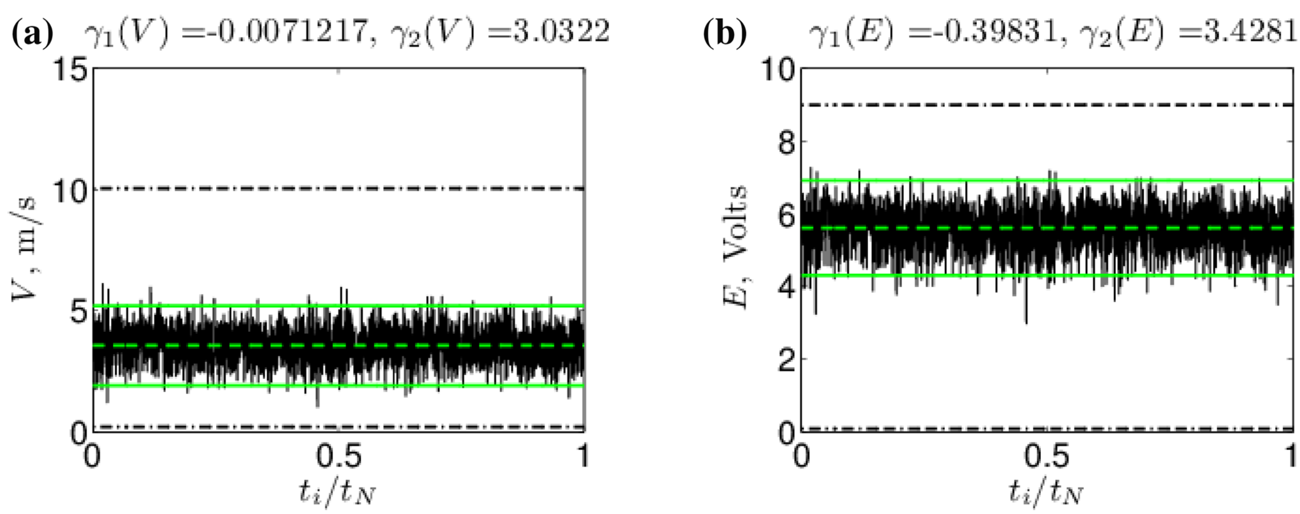

Fig. 5 **a** Calibrated hot-wire signal $V(t)$ and **b** measured voltage signal $E(t)$ obtained at $(x, y, z)/a_c = (3.75, 0.25, -0.25)$ and $U_\infty = 5$ m/s. t_i is a discrete time sample out of N samples. The *green broken lines* are the mean values, each of the *solid green lines* represents an offset of 2.5σ from its mean value, and the *black dash-dot lines* are boundaries of the calibration range (shown in Fig. 1)

m/s, for which the corresponding Reynolds numbers are $Re_a = 8.9 \times 10^3$ and $Re_a = 1.15 \times 10^4$, respectively.

The desired signal properties for testing the new calibration method are obtained by placing the hot-wire probe at $a_c/4$ distance from the cube upper and side faces, i.e., at $z, y = \pm 0.25a_c$ on the y–z plane at a certain downstream location where $x > 3a_c$. In Fig. 4, we plot the variation of the first four statistical moments of the measured velocity V along the downstream direction of the wake for $(y, z) = (0.25a, -0.25a)$ and $U_\infty = 5$ m/s. It is evident that for $x > 3a_c$, the values of skewness and kurtosis are close to the Gaussian ones.

To apply our method of calibration, we place the sensor at a downstream location $x = 3.75a_c$ and $y = 0.25a_c$, $z = -0.25a_c$. The measured velocity signal at this location is shown in Fig. 5a. The corresponding measured voltage signal is shown in Fig. 5b. This output voltage signal, along with two measured calibration data points (E_1, V_1), (E_2, V_2) (marked by red circles in Fig. 6), is used to estimate the hot-wire calibration curve using the method of Gaussianization.

The obtained signal Z and the estimated result, denoted as $\widehat{V}$, are displayed in Fig. 6. In Fig. 7, we present the pdfs of the measured signals $V(t)$ and $E(t)$, together with the pdf of the corresponding signal $Z(t)$ obtained by Eq. (2) and the pdf of the estimated signal $\widehat{V}$ obtained by Eq. (3). From Fig. 6, it can be seen that the method preforms very well, and successfully estimates the calibration curve of the hot-wire sensor in the region of the measured signal, which appears to be almost Gaussian as can be seen from Fig. 7. Similar results have been obtained for other velocity ranges and locations in the wake where the values of skewness and kurtosis of the velocity signal are almost equal to the Gaussian ones.

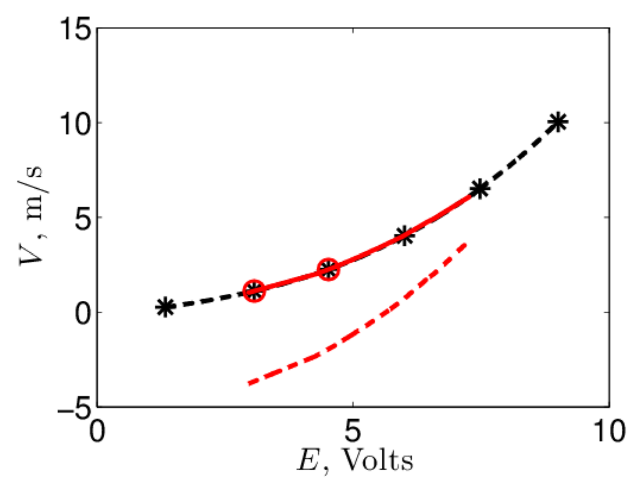

Fig. 6 Estimated velocity calibration curve $\widehat{V}$ (*red solid line*) vs. calibration curve obtained by the standard method (*black dashed line*) based on the measured data points (*asterisks*). The two provided calibration data points are marked by the *red circles*. The obtained signal Z is shown by the *red dashed line*

For homogeneous turbulence, it is noted in Jimenez (1998) that the CLT does not necessarily hold, i.e., the probability distributions of the components of the turbulent velocity fluctuations are not necessarily Gaussian. Thus, the new method's sensitivity to the distribution assumption is studied next.

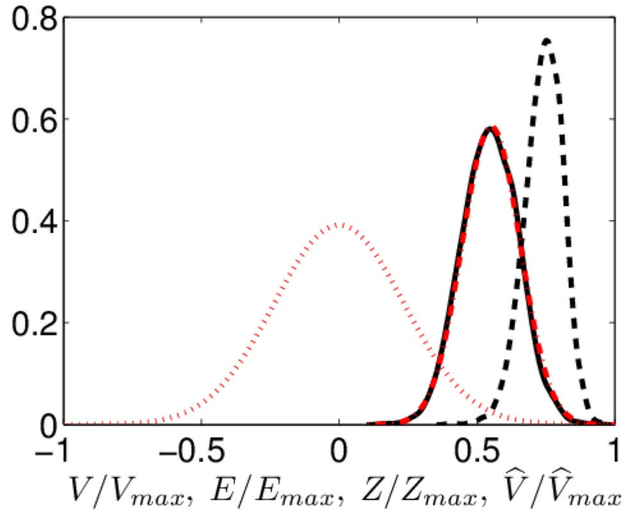

Fig. 7 Pdf's of V (*black solid line*), E (*black dashed line*), Z (*red dotted line*), and $\widehat{V}$ (*red dash-dot line*), measured at $(x, y, z)/a_c = (3.75, 0.25, -0.25)$ and $U_\infty = 5$ m/s

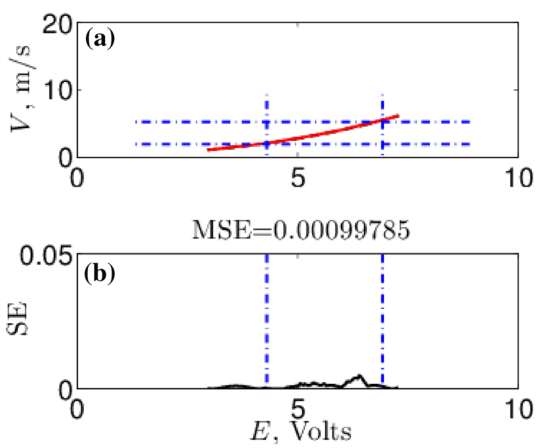

Fig. 8 **a** Plot of $V(E)$ (*black dashed line*) and $\widehat{V}(E)$ (*red solid line*). **b** Corresponding SE. *Blue dashed lines* indicate $\pm 2.5\sigma$ bounds for V (*horizontal lines*) and E (*vertical lines*). $(x, y, z)/a_c = (3.75, 0.25, -0.25)$, $U_\infty = 5$ m/s

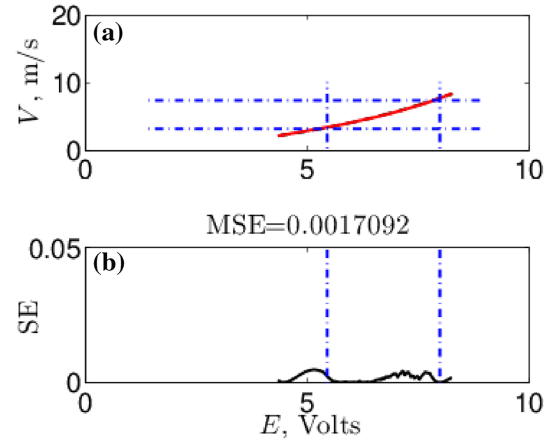

Fig. 9 **a** Plot of $V(E)$ (*black dashed line*) and $\widehat{V}(E)$ (*red solid line*). **b** Corresponding SE. *Blue dashed lines* indicate $\pm 2.5\sigma$ bounds for V (*horizontal lines*) and E (*vertical lines*). $(x, y, z)/a_c = (4.25, 0.25, -0.25)$, $U_\infty = 7$ m/s

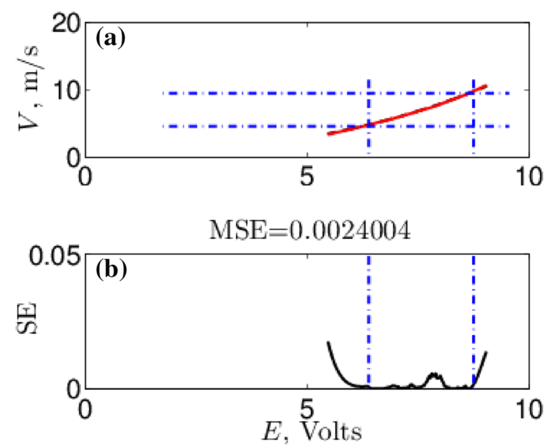

Fig. 10 **a** Plot of $V(E)$ (*black dashed line*) and $\widehat{V}(E)$ (*red solid line*). **b** Corresponding SE. *Blue dashed lines* indicate $\pm 2.5\sigma$ bounds for V (*horizontal lines*) and E (*vertical lines*). $(x, y, z)/a_c = (4.75, 0.25, -0.25)$, $U_\infty = 9$ m/s

4 Robustness to method assumptions

In this section we study the sensitivity of the proposed method to the assumption that the turbulent flow velocity signal is normally distributed. In addition, we study the difference between the results obtained by our method and those obtained using the traditional methods.

The estimation error between $V = g(E)$ and $\widehat{V} = \widehat{g}(E)$ is measured by the mean-squared error (MSE), defined as follows:

$$\text{MSE} \triangleq \frac{1}{m - l + 1} \sum_{i=l}^{m} \text{SE}_i, \tag{9}$$

where SE_i is a local squared error, that is

$$\text{SE}_i = \left[g(E_i) - \widehat{g}(E_i)\right]^2, \quad i = l, \ldots, m, \tag{10}$$

and $E_l = \min[E(t)]$, $E_m = \max[E(t)]$.

In Figs. 8, 9, and 10, we show SE and MSE of the obtained $\widehat{V}$ for $U_\infty = 5, 7, 9$ m/s, respectively, at $(y, z)/a_c = (0.25, -0.25)$ at a selected downstream location x determined by the smallest MSE. We observe that the edge regions off the $\widehat{V}$ curve contribute to the most to the MSE. This is attributed to the lack of samples in these regions which corresponds to the tails of the pdf of the signal $V(t)$.

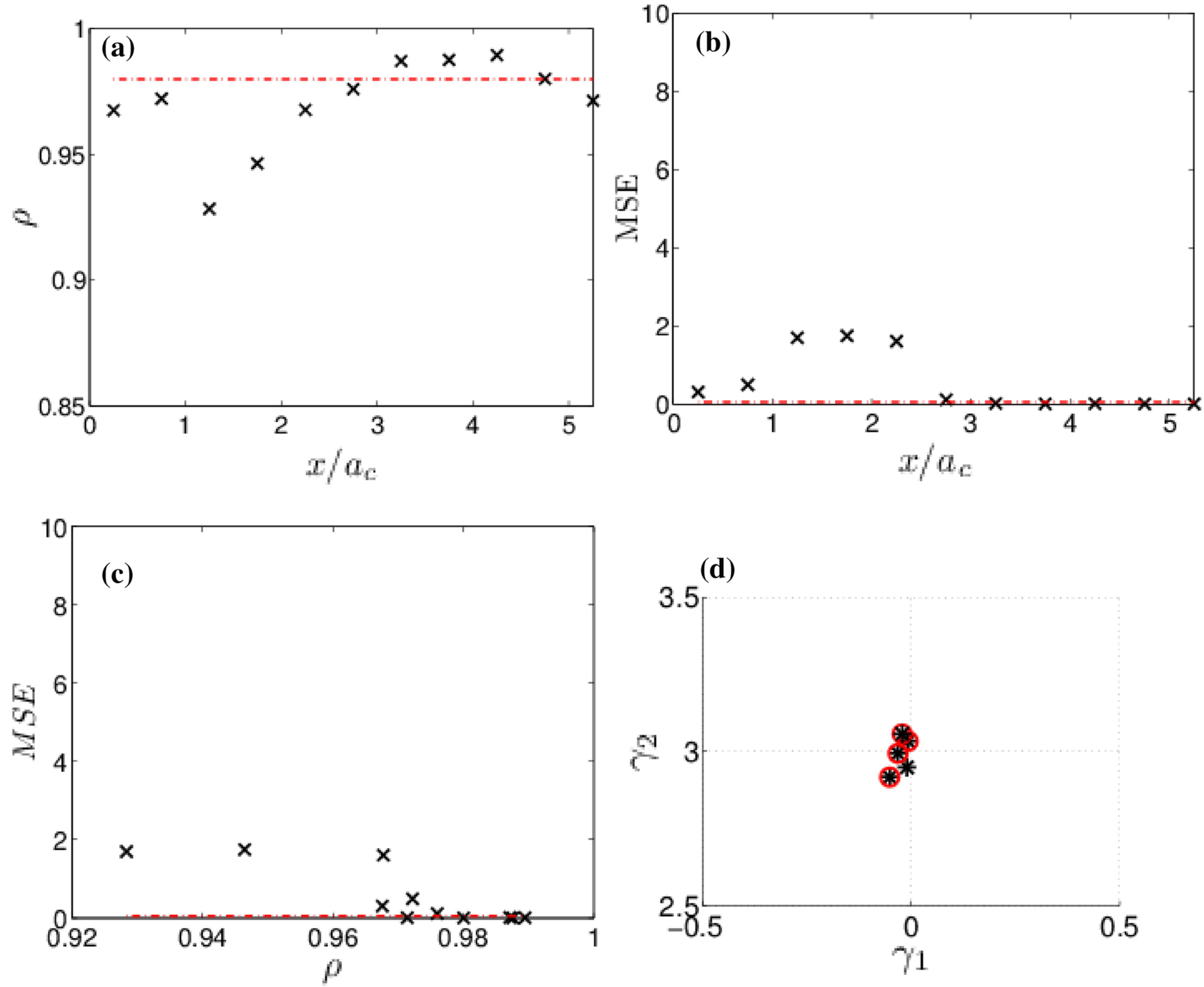

Fig. 11 **a** Variation of ρ along the streamwise direction ($\rho = 0.98$: *red dashed line*). **b** Variation of MSE along the streamwise direction (MSE = 0.05: *red dashed line*). **c** MSE vs. ρ. **d** MSE as function of $\gamma_1(V)$ and $\gamma_2(V)$. (MSE < 0.05: *black asterisks*, MSE < 0.01: *red circles*). All signals measured at $y = 0.25a_c$, $z = -0.25a_c$, and $U_\infty = 5$ m/s

It can be seen that the method performs well in the bounded range $\mu_V \pm 2.5\sigma_V$ of the signal $V(t)$, in which MSE < 0.01 for the three freestream velocities tested.

In addition, because Z is linearly related to $\widehat{V}$ (Eq. (3)), we test the quality of the estimation by evaluating the correlation coefficient ρ between Z and V,

$$\rho \triangleq \frac{\mathrm{cov}(V, Z)}{\sigma_V \sigma_Z}, \quad |\rho| \leq 1. \tag{11}$$

In each of Figs. 11, 12, and 13, we plot 4 charts: charts (a) and (b) show the variations of ρ and MSE, respectively, along the wake for $(y, z) = (0.25a, -0.25a)$ on the y–z plane for freestream velocities of $U_\infty = 5, 7, 9$ m/s. From these charts, we see that signals obtained at downstream distances $x > 3a_c$ are characterized by $\rho > 0.98$ and MSE < 0.05, demonstrating the good performance of our calibration method. Chart (c) shows the relation between MSE and ρ. It can be seen that, in general, as ρ approaches 1, the MSE tends to 0, except for some localized regions.

To study the sensitivity of the proposed method to the assumption that the turbulent flow velocity signal is normally distributed, we study the effect of the skewness, $\gamma_1(V(t))$, and kurtosis, $\gamma_2(V(t))$, of the velocity signal $V(t)$ on the performance of the method. The fourth chart (d) in each of Figs. 11, 12, and 13 shows the MSE values obtained for specific skewness and kurtosis values of the measured velocity V signals obtained at $(y, z) = (0.25a, -0.25a)$ on the y–z plane for different downstream locations. It is shown that MSE < 0.05 is obtained for skewness and kurtosis values close to their Gaussian values within deviations

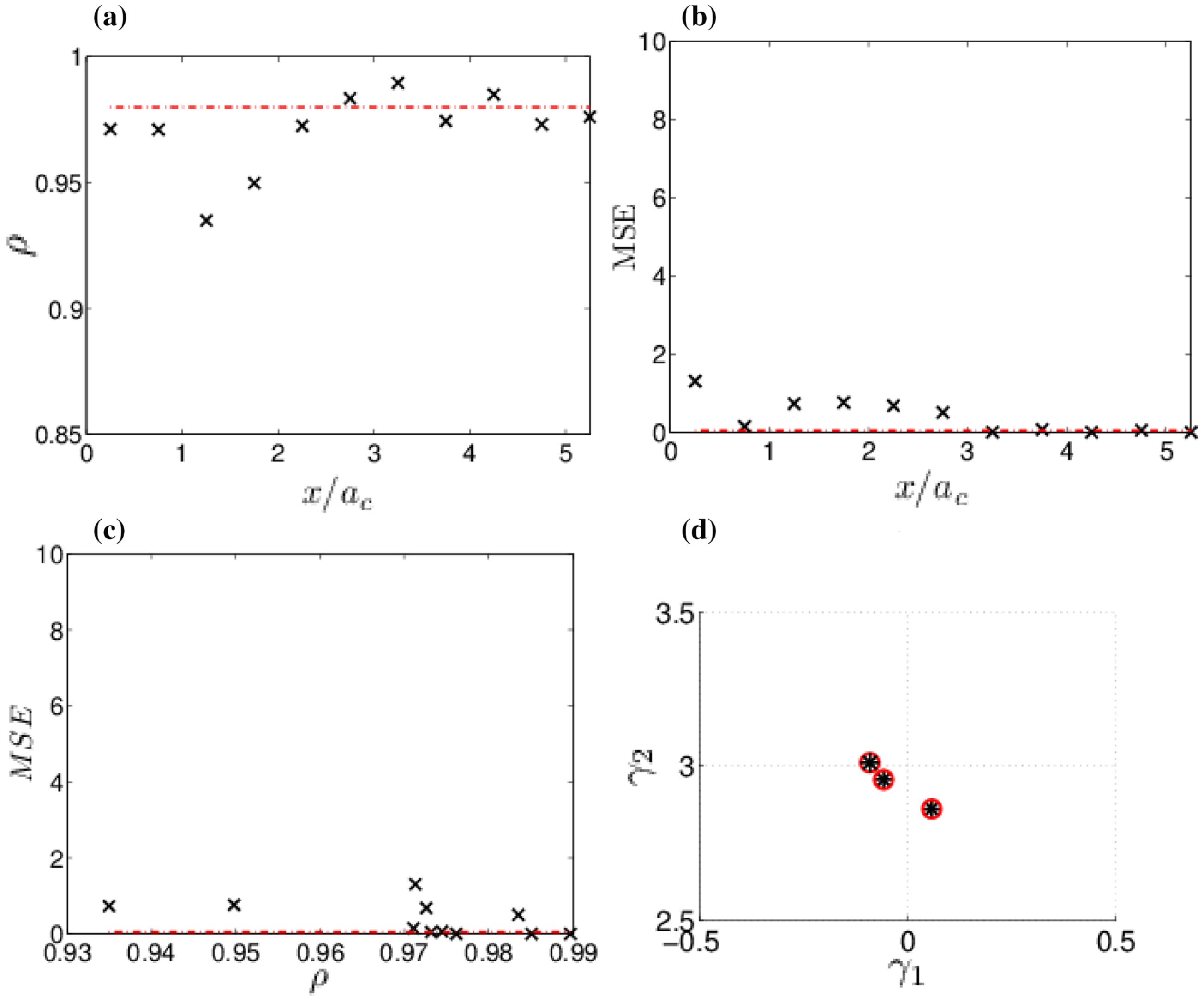

Fig. 12 **a** Variation of ρ along the streamwise direction ($\rho = 0.98$: *red dashed line*). **b** Variation of MSE along the streamwise direction (MSE = 0.05: *red dashed line*). **c** MSE vs. ρ. **d** MSE as function of $\gamma_1(V)$ and $\gamma_2(V)$. (MSE < 0.05: *black asterisks*, MSE < 0.01: *red circles*). All signals measured at $y = 0.25a_c$, $z = -0.25a_c$ and $U_\infty = 7$ m/s

of approximately ±0.15. These values are sufficient to generate a nearly Gaussian distribution which is sufficient to estimate *Z*. For larger deviations of γ_1 and γ_2, the performance of the method degrades.

5 Concluding remarks

We have presented a statistical method for recovering the nonlinear relation between the input velocity and the output voltage of a hot-wire sensor. The method uses as input a measured sequence of voltage samples, corresponding to unknown flow velocities in the desired operational range, together with only two measured voltages along with their known associated flow velocities. In addition to the calibration obtained over the range spanned between the two calibration data points, the new method can provide an extended range of calibration, covering velocities below and above the calibration data points. To this end, a velocity signal having a known distribution has to be provided that covers the extended velocity range. The method is based on a Gaussianization technique which works very well if the velocity signal is normally distributed. Normal velocity distributions can be achieved if the hot-wire sensor is placed in a turbulent flow regime under certain conditions. Therefore, the method relies on the spatial resolution of the probe and the existence of sufficiently many data samples, in order to obtain a converged pdf of the measured signal.

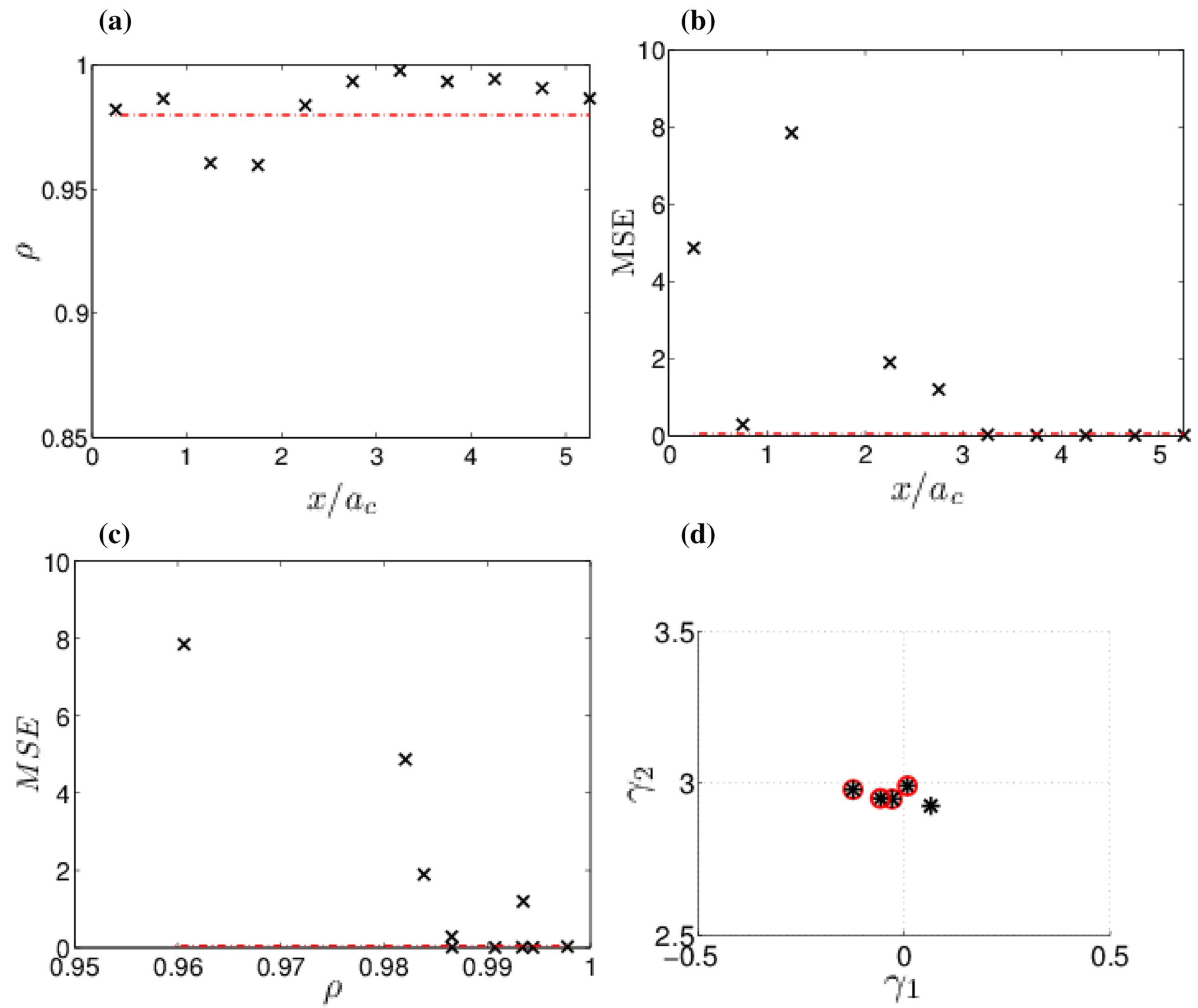

Fig. 13 **a** Variation of ρ along the streamwise direction ($\rho = 0.98$: *red dashed line*). **b** Variation of MSE along the streamwise direction (MSE = 0.05: *red dashed line*). **c** MSE vs. ρ. **d** MSE as function of $\gamma_1(V)$ and $\gamma_2(V)$. (MSE < 0.05: *black asterisks*, MSE < 0.01: *red circles*). All signals measured at $y = 0.25a_c$, $z = -0.25a_c$ and $U_\infty = 9$ m/s

It has been demonstrated that the method performs well even for signals that are only approximately Gaussian, as long as the deviations from the assumed Gaussian distribution are not too large. In general, the method can be easily modified to accommodate distributions other than Gaussian and will perform well if the flow distribution is known.

The cube wake, which has been used in this paper, is a good candidate for demonstrating the performance of the proposed method in a real flow case, but this configuration is by no means unique, and one can consider any other geometry of a bluff body with different surface roughness under varying flow regimes (Reynolds numbers). We have shown that varying the Reynolds numbers of the cube-shaped body in the range of 6.4×10^3–1.15×10^4 does not affect the statistical distributions required for applying the method, provided that the hot-wire probe is placed at a downstream distance larger than 3 diameters of the cube. The method can be directly applied in many flow cases, e.g., turbulent wakes of bluff bodies, such as cubes and spheres, turbulent flow past a grid, or any flow for which it is possible to obtain consistent known pdfs of the turbulent velocity signal.

Finally, to estimate the nonlinear mapping of the hot-wire sensor by the proposed method in a certain velocity range, the recorded velocity signal should cover this range. Thus, to increase the calibration range, the velocity input signal should be highly turbulent, with large velocity fluctuations.

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