Selective Modal Transducers for Anisotropic Rectangular Plates: Experimental Validation

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A general selective modal transducer (SMT) design methodology recently introduced by the authors for piezolaminated anisotropic plate systems is validated through an experimental test on a cantilevered orthotropic composite piezolaminated plate. Fundamental aspects of the SMT theory are reviewed. The SMT theory is extended to provide the means to predict the modal character of multilayered piezolaminated transducers embedded in an anisotropic plate in which the active subelements are both bidirectionally and spatially varying. An experimental procedure is described involving a 10-layered orthotropic plate constructed from four graphite-epoxy layers sandwiched between six piezoelectrically active polyvinylidene fluoride (PVDF) sublaminates. Three PVDF sublaminates stacked on one face are combined electrically to provide a single sensed measurement, while the three remaining PVDF sublaminates stacked on the opposing face are combined to provide single channel actuation. Lead compensation is employed to provide active control. Open- and closed-loop frequency and transient response data are then analyzed, and significant active vibration attenuation is observed. A numerical simulation directly based on an SMT-derived transducer model is developed, and simulation results are compared with observed system behavior. The theoretically based results closely resemble the measured response to within an expected range of accuracy, thus validating the transducer theoretical model predicted by the SMT theory.

I. Introduction

Within the past decade, several vibration control techniques have been developed for simple beam and plate systems that utilize distributed piezoelectric transducers formed from polyvinylidene fluoride (PVDF). PVDF actuators have been designed whose spatially varying piezoelectric field properties were exploited to provide for the simultaneous control of all modes or special modal subsets in cantilevered and simply supported beams. 1, 2 Miller and Hubbard 3 developed a reciprocal sensor theory and subsequently incorporated PVDF sensors and actuators into multicomponent systems in which each component itself was a smart structural member. Burke and Hubbard 4 developed a formulation for the control of thin elastic (Kirchhoff–Love) isotropic plates subject to most combinations of free, clamped, or pinned boundary conditions, in which the active elements were spatially varying biaxially polarized piezoelectric transducer layers. Lee 5 generalized the classical laminate plate theory to include the effect of laminated piezoelectric layers and thus to provide a theoretical framework for the distributed transduction of bending, torsion, shearing, shrinking, and stretching in flexible anisotropic plates. Miller et al. 6 subsequently employed Lyapunov’s second method to derive a general active vibration suppression control design methodology for anisotropic laminated piezoelectric plates.

The aforementioned vibration control strategies share several common limitations. Perhaps most significantly, although all of these methods reduce the vibration control task to a selection of individual piezolamine field functions, none offers a general method for determining those field functions so as to ensure active vibration suppression. Therefore a clearly defined design methodology was developed by the authors 7, 8 in which the structural and control design processes are integrated to yield a framework for truly selective modal control (SMC). The SMC approach requires the implementation of selective modal transducers (SMTs) that utilize multiple piezolamine layers to provide modal transduction based on a selected choice of modal participation factors (MPFs). A broad class of stability robust SMC approaches was defined through the identification of conditions that sufficiently ensure global asymptotic stability without requiring perfect knowledge of design parameters, structural constants, or modal behavior. Specific SMC design examples have been presented that allow for 1) the contributions of any given mode to the active energy extraction rate to be directly specified, 2) pole locations to be selectively and dynamically varied, or else 3) pole locations, SMT design parameters, and feedback laws to be optimally determined.

The SMC approach is contingent on the successful development of SMTs via proper implementation of the SMT theory. Although several numerical examples have been presented that support both the SMT and SMC theories, the present study is primarily concerned with demonstrating the SMT concept on actual hardware. A secondary objective is to describe a means through which the SMT theory may be used in the converse sense to predict the MPFs associated with a given multilayered transducer and known piezospatial variance properties. To achieve these goals, a 10-layered skew symmetric orthotropic plate was constructed, consisting of four graphite-epoxy layers embedded between two SMTs. Each SMT contains three PVDF piezolamine layers and assumes either a pure sensing or pure actuation function. A simple lead compensation circuit that emulates a stability robust control law developed in Ref. 7 at low frequencies was constructed. The structure was cantilevered and excited externally through an electromagnetic coil. Open- and closed-loop frequency and transient response data were obtained for the first four structural modes. Using the SMT theory, the actual system behavior was predicted through numerical simulation. A comparison was then made between actual and predicted dynamic behavior, which effectively validates the SMC theory.

II. System Description

Figure 1 describes the geometry of the general system under consideration. A rectangular anisotropic plate with exactly N piezoelectrically active laminated layers is considered. Contrary to what may be implicitly assumed from Fig. 1, transducer layers may be located anywhere in the structure. The length dimensions of the system in
the x and y directions are denoted as \( L_x \) and \( L_y \), respectively. Each piezoelectric layer may be independently anisotropic, and its electromechanical field strength may be selectively varied in both spatial dimensions. The material properties within each lamina are assumed continuous. A complete system description may be found in Ref. 8.

Applying the well-known Kirchhoff–Love approximation, the authors developed the equations of motion of the general system described in Fig. 1 in Ref. 8, and the equations are expressed in the form

\[
x_t + C_x + \mathbf{K} = -\frac{1}{\rho \mathbf{D}^T} \left( \sum_{k=1}^{N} \mathbf{e}_k \mathbf{A}_k \mathbf{V}^k \right)
\]

(1)

where a subscript \( t \) indicates partial temporal differentiation,

\[
\mathbf{D}, \mathbf{C}, \mathbf{K} = \begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 & 0 & 0 & 0 \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2}{\partial x^2} + 2 \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2} & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2}{\partial x^2} + 2 \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2} & 0 & 0 & 0 \\

\end{bmatrix}
\]

(2)

and

\[
\mathbf{X} = [\mathbf{x}(x, y), \mathbf{v}(x, y), \mathbf{w}(x, y)]^T
\]

\[
\mathbf{K} = (1/\rho \mathbf{D}^T) \mathbf{D} \mathbf{K}, \quad C = bo I + co \mathbf{K}
\]

(3)

The term \( \mathbf{K} \in \mathbb{R}^{6 \times 6} \) is the standard material stiffness matrix containing the usual \( A_{ij}, B_{ij}, \) and \( D_{ij} \) constitutive constants that characterize the mechanical stress–strain behavior of the composite system\(^2\), and \( u, v, \) and \( w \), respectively, the \( x, y, \) and \( z \) directional displacements. The constants \( b_0 \) and \( c_0 \) are viscous and structural damping coefficients, respectively, whereas \( I \) is the identity operator. The term \( V^k \) is the spatially independent driving voltage applied across the \( k \)th piezolamina. The electromechanical field strength of each piezolamina is described mathematically via the product \( \mathbf{e}_k \mathbf{A}_k \), where \( \mathbf{A} \) is a dimensionless and spatially varying piezoelectric field distribution function (PFF). The vector \( \mathbf{e}_k \) is defined as \( \mathbf{e}_k = [\mathbf{e}_k^T \mathbf{z}_k^* \mathbf{e}_k^T] \), where \( \mathbf{e}_k \in \mathbb{R}^3 \) is a vector of piezoelectric constants relative to the point of maximum electromechanical transduction.

The boundary conditions as derived in Ref. 8 are stated in Table 1, where the in-plane resultant forces \( \mathbf{N} \equiv [N_x N_y N_z]^T \) and moments \( \mathbf{M} \equiv [M_x M_y M_z]^T \) are given by

\[
\mathbf{N} = -\mathbf{K} \mathbf{X} - \sum_{k=1}^{N} \mathbf{e}_k \mathbf{A}_k \mathbf{V}^k
\]

(4)

Theorem 1: Consider an anisotropic rectangular plate containing \( N \) piezolaminas whose equations of motion are given by Eq. (1). Assume each piezolamina functions as a self-sensing actuator. Let
the time bound control input $V^k(t)$ of each piezolamina be proportional to an identical time-dependent control function $V_n(t)$ such that $V^k(t) = g_0^k V_n(t)$. Let the measured state $i_k(t)$ formed from the weighted sum of the sensed currents of each individual lamina be such that

$$i_k(t) = \sum_{j=1}^{N} g_0^j i^j_k(t)$$

Assume that conditions C1–C5 are satisfied. Then, if the piezoelectric field distribution functions of each active layer are given by

$$\Lambda^k = (1/\Phi^0_k) (\hat{e}^0_k)^T R^{-1} K, \tilde{\Phi}(x, y) \ \ (10)$$

where $\hat{\Phi}$ is defined in Eq. (9), $R$ is defined in Eq. (8), and $g_0$ is defined as

$$g_0^k = \max_{(x, y) \in \chi} \left| (\hat{e}^0_k)^T R^{-1} K, \tilde{\Phi}(x, y) \right| \ \ (11)$$

the equations of motion of the plate reduce to the form

$$\tilde{q}_m + (b_0 + c_0 \lambda^m + \lambda_0 q_m = -\alpha_m \lambda^m V_n(t) \ \ (12)$$

whereas the measured state reduces to the form

$$i_k(t) = - \rho h \sum_{j=1}^{N} \alpha_j \lambda_j q_j(t) \ \ (13)$$

for all integers $m > 0$, where $\alpha_m, \lambda_m$, and $q_m(t)$ are, respectively, the modal participation factor, eigenvalue, and generalized modal coordinate associated with the $m$th mode.

Should voltage rather than current be preferred as the measured state so that Eq. (7) replaces Eq. (6), then

$$V_j(t) = - \rho h \sum_{j=1}^{N} \alpha_j \lambda_j q_j(t) \ \ (14)$$

should replace Eq. (13) (Ref. 11).

Theorem 1 represents an anisotropic piezolaminated plate selective modal actuator and/or sensor design methodology. The SMT construct conditions C1–C5 are obeyed. Mode shapes belonging to the target modal subset are determined and then assigned modal participation factors. Then, using Eq. (10) as an algorithm for determining the piezoelectric field function for each layer and enforcing the condition $V^k(t) = g_0^k V_n(t)$, a modal actuator design is realized that is capable of exciting each mode according to its relative weighting. If instead the condition

$$\tilde{q}_m = 0$$

is enforced, then a selectively weighted modal measurement may be obtained.

If the composite system is orthotropic so that there is no inherent mechanical coupling between bending and stretching motions, then the differential operators $D$ and $E$ simplify:

$$R = \sum_{k=1}^{N} \tilde{e}^0_k (\hat{e}^0_k)^T$$

replaces its previous definition in Lemma 1, and the following two relaxed conditions replace the SMT construct conditions C1–C5 (Ref. 8).

C6: There are at least three piezoelectrically active layers ($N \geq 3$).

C7: The piezoproperty vectors $\{\tilde{e}^0_k\}_{k=1}^{N}$ associated with at least three layers are different. When the same piezostack material is used throughout, $\tilde{e}^0_1(\Theta = 0 \ deg) \neq \tilde{e}^0_2(\Theta = 0 \ deg)$, and the skew angles of at least three laminae must be different in the range $-90 \degree < \Theta < 90 \degree$.

IV. Nonselective Modal Transducers (NSMTs)

It is often preferable to develop transducers in which the PFF $\Lambda^k(x, y)$ of each layer is chosen on the basis of facilitating practical implementation rather than selectively assigning modal participation factors to each structural mode. Hence the transducers that result are referred to as NSMTs. However, by expanding the scope of Theorem 1, the set of modal participation factors that correspond to any given NSMT may be predetermined and thus its performance evaluated in the course of synthesizing an appropriate design. The following theorem formalizes the NSMT concept.

**Theorem 2:** Consider an anisotropic (Kirchhoff–Love) thin plate containing $N$ piezolaminates whose equations of motion are given by Eq. (1). Assume that each lamina is to function as a self-sensing actuator such that the sensed measurement of the $k$th layer is given by Eq. (13). Assume that conditions C1–C5 are satisfied. Assume also that piezolaminates gain constants $[g_0^k]_{k=1}^{N}$ and field functions $(\Lambda^k)_{k=1}^{N}$ are known. Let the measured state $i_k(t)$ formed from the weighted sum of the sensed currents of each individual lamina be such that

$$i_k(t) = \sum_{j=1}^{N} g_0^j i^j_k(t)$$

Let the time bound control input $V^k(t)$ of each piezolamina be proportional to an identical time-dependent control function $V_n(t)$ such that $V^k(t) = g_0^k V_n(t)$. Then the equations of motion and measured state are, respectively, reduced to Eqs. (12) and (13) for all integers $j > 0$, where the modal participation factors $\alpha_j$ are given by

$$\alpha_j = \frac{1}{\rho h} \lambda_j \hat{\Phi}_j, \sum_{k=1}^{N} g_0^k \hat{e}^0_k \Lambda^k = \frac{1}{\rho h} \sum_{k=1}^{N} g_0^k \hat{e}^0_k \Lambda^k \ \ (15)$$

Remark: As with Theorem 1, conditions C6 and C7 replace conditions C1–C5 when Theorem 2 is applied to orthotropic systems.

Proof: Theorems 1 and 2 both require that conditions C1–C5 are satisfied. Moreover, in both theorems the actuator and sensor equations are, respectively, given by Eqs. (12) and (13). In Theorem 1, however, each PFF $\Lambda^k$ is determined as a function of known MPFs, whereas in Theorem 2 the MPFs are determined from known PFFs. Consequently, Theorem 2 may be proven directly from Theorem 1 through determining which set of MPFs yields a specific set of PFFs. Premultiplying the left side of Eq. (10) by $g_0^k \hat{e}^0_k$, summing over $N$ layers, applying Lemma 1, and recalling that

$$\hat{\Phi} = \sum_{k=1}^{N} \alpha_k \Phi_k, \sum_{k=1}^{N} g_0^k \hat{e}^0_k \Lambda^k = \sum_{k=1}^{N} \alpha_k K \tilde{\Phi}_j, \ \ (16)$$

yields

$$\sum_{k=1}^{N} g_0^k \hat{e}^0_k \Lambda^k = \sum_{k=1}^{N} \alpha_k K \tilde{\Phi}_j$$

Relying on the identity $(\tilde{\Phi}_j, K, \tilde{\Phi}_j) = \rho h \lambda_j \delta_j$ established in Ref. 11,

$$\tilde{\Phi}_j, \sum_{k=1}^{N} g_0^k \hat{e}^0_k \Lambda^k = \rho h \alpha_j \lambda_j \delta_j \ \ (17)$$

from which Eq. (15) follows trivially.

V. Experimental Validation

The SMT theory was validated through experimental test on a cantilevered orthotropic composite piezolaminated plate. However, resources were not available for selectively doping piezoelectric copolymer film with a secondary material so as to realize the required PFFs. Instead, commercially available uniformly piezoelectric PVDF was obtained and a single nonselective modal actuator (NSMA) and nonselective modal sensor (NSMS) were designed and then incorporated into the structure. Using Theorem 2, theoretical predictions for the MPFs corresponding to the NSMTs were determined. The resulting NSMTs then become synonymous with SMTs whose MPFs are those that were predicted. Emulating rate feedback of the NSMS measured voltage [Eq. (14)] output through a lead compensation
network, the structure was actively controlled and the closed-loop damping coefficients of the first four vibrational modes were obtained. The rate feedback law was selected because, when ideally implemented, it inherently guarantees stability robustness regardless of piezospatial variance. Significant active vibration attenuation was observed. A numerical simulation of the system was then derived and compared with the actual system behavior. The theoretical results were seen to match the measured response within an expected range of accuracy, thus substantiating the SMT theory.

A. Test Structure Design
The test structure design is illustrated through the side and exploded views given in Fig. 2. The aspect ratio of the plate, whose dimensions are given in Fig. 2, is approximately two. The horizontal and vertical coordinates, respectively, are referred to as $x$ and $y$. Three PVDF layers were laminated to each face of a 4-layered graphite-epoxy substrate, forming a 10-layered specimen. A schematic of the test bed configuration (top view) is given in Fig. 3. The structure was driven externally through an electromagnetic coil. The structure was then cantilevered, and 17 gram masses were attached as close as possible to the free corners both to stabilize the structure and to allow for the structure to be externally driven by an electromagnet.

B. Experimental Procedure
The test bed configuration is shown schematically in Fig. 3. The structure was driven externally through an electromagnetic coil. The theoretical results were seen to match the measured response within an expected range of accuracy, thus substantiating the SMT theory.

### Table 2

<table>
<thead>
<tr>
<th>Property</th>
<th>Layers 1, 10</th>
<th>Layers 2, 9</th>
<th>Layers 3, 8</th>
<th>Layers 4, 7</th>
<th>Layers 5, 6</th>
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<td>$E_{11}$, GPa</td>
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<td>2.00</td>
<td>2.00</td>
<td>14.5</td>
<td>14.5</td>
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<tr>
<td>$E_{22}$, GPa</td>
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<td>2.00</td>
<td>2.00</td>
<td>9.60</td>
<td>9.60</td>
</tr>
<tr>
<td>$G_{12}$, GPa</td>
<td>1.42</td>
<td>1.42</td>
<td>1.42</td>
<td>4.10</td>
<td>4.10</td>
</tr>
<tr>
<td>$\rho$, kg/m$^2$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Thickness, μm</td>
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<td>1,780</td>
<td>1,551</td>
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<tr>
<td>$(e_{1})_o=-\phi$, C/m$^2$</td>
<td>60,000</td>
<td>60,000</td>
<td>60,000</td>
<td>137.5</td>
<td>137.5</td>
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<tr>
<td>$(e_{2})_o=-\phi$, C/m$^2$</td>
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<td>20,000</td>
<td>20,000</td>
<td>22.5</td>
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<td>0</td>
<td>$-60$</td>
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Fig. 2 Test structure, side and exploded views.

Fig. 3 Schematic of test bed configuration (top view).
Table 3 Damping coefficients and natural frequencies: experimental structure

<table>
<thead>
<tr>
<th>Mode</th>
<th>Open loop</th>
<th>Theory</th>
<th>Experiment</th>
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<tr>
<td></td>
<td>(\gamma_n)</td>
<td>(f_n), Hz</td>
<td>(\gamma_n)</td>
</tr>
<tr>
<td>1</td>
<td>0.0245</td>
<td>9.2344</td>
<td>0.0657</td>
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<td>2</td>
<td>0.0342</td>
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<td>0.1135</td>
</tr>
<tr>
<td>3</td>
<td>0.0413</td>
<td>80.370</td>
<td>0.0562</td>
</tr>
<tr>
<td>4</td>
<td>0.0465</td>
<td>127.62</td>
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</tr>
</tbody>
</table>

Voltage measured across the NSMS terminals, \(V_s(t)\), was allowed as an input to a controller block. The controller block was realized through the simple analog circuit cascaded with a high-voltage amplifier. The output of the control block, \(V_a(t)\), then drives the NSMA when the switch is closed. Both \(V_a\) and the coil drive voltage \(V_c\) were processed through a Hewlett-Packard HP5660A spectrum analyzer to acquire the experimental data. The controller, whose transfer function is given in Fig. 3, causes \(V_a(t)\) to be nearly proportional to the measured current \(i_s(t)\) at low frequencies.

Three experimental test procedures were conducted. In the first procedure, frequency response data relating \(V_s\) to the drive coil voltage \(V_c\) were obtained in both the open- and closed-loop scenarios. Damping coefficients and resonant frequencies of the first four modes were then determined. A free decay test was then performed: the structure was excited via the drive coil at a modal frequency until a steady-state response was obtained, and then the excitation was removed. Open- and closed-loop time histories were thus obtained for the first three structural modes. Last, the structure was subject to continuous excitation at a resonance frequency with the controller off, and the time history was recorded as the controller was turned on. The latter procedures served to demonstrate controller performance, whereas the frequency response test provided damping coefficient and resonance frequency data necessary to validate the theoretical model.

C. Experimental Results

Frequency Response
Free Decay Test
Free decay test results for the first three modes are given in Figs. 5–7. The instant in which the sinusoidal excitation to the drive coil is turned off is indicated in each time history by a vertical dashed line. In all cases, significant active vibration suppression is observed. In the second mode response (Fig. 6), removal of the input tended to marginally excite first mode oscillations as well. The high-frequency ripple in the second and third mode controlled responses is attributed to 50-Hz line noise that entered the system through the control electronics.

Continuous Excitation Test
First, second, and third mode continuous excitation test results are shown in Fig. 8. The introduction of active control is indicated by the vertical dashed line in both time histories. A significant reduction in the amplitude of the vibrational response of all three modes is observed when the controller is turned on.

D. Theoretical Results
Applying Theorem 2, the authors determined the MPFs corresponding to the transducers implemented on the experimental structure. The NSMT themselves may then be thought of as SMTs whose MPFs are those determined via Theorem 2. The experimental structure satisfies both orthotropic plate construct conditions C6 and C7. For orthotropic plates in bending, Eq. (15) is reduced to the form

\[
\alpha_i = \frac{1}{\rho h \Lambda_i} \left( \mathcal{E} \phi_i^T, \sum_{j=1}^{N} \mathcal{H}_j \Lambda_j \right)
\]

where

\[
\mathcal{E} = - \left[ \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} 2 - \frac{\partial^2}{\partial x \partial y} \right]^T
\]

based on simplifications found in Ref. 8. The \(i\)th out-of-plane bending mode is referred to as \(\phi_i[w(x, y)]\). In the experimental structure all \(\Lambda_i = 1\), whereas the electrical attachments and layer polarities ensure that \((g_1, g_2, g_3) = (g_1, g_2, g_3) = (1, -1, 1)\). Hence the MPFs identified with the experimental structure may be readily calculated once mode shapes are determined.

A discrete model of the experimental structure was developed to obtain high-fidelity mode shape approximations. The ANSYS finite element modeling (FEM) package was used to generate mass and stiffness matrices \((M, K)\). The eigenfunctions obtained through
the FEM analysis, the first four of which are shown in Fig. 9, were then best fit (in a least-squares sense) to a polynomial of the form

\[ \Phi_j = [x^6 \ x^5 \ x^4 \ x^3 \ x^2 \ x \ 1] \cdot P[y^6 \ y^5 \ y^4 \ y^3 \ y^2 \ y \ 1]^T \]

(20)

where \( P \in \mathbb{R}^{7 \times 7} \) is a polynomial coefficient matrix. Boundary constraints were included in the determination of \( P \) to improve the fidelity of the approximation (see Sec. V.E). Equation (20) was then used in conjunction with Eq. (18) to determine the MPFs corresponding to the first 10 structural modes. Errors associated with FEM open-loop eigenvalue predictions were essentially eliminated through setting the first four open-loop natural frequencies and damping factors to experimentally observed values.

To theoretically predict the closed-loop damping coefficients and natural frequencies, the (open-loop) system was represented in the modally decomposed form

\[ \ddot{q} + \bar{C} \dot{q} + \bar{K} \dot{q} = L(t) \]

(21)

Fig. 5  First mode free decay test results.

Fig. 6  Second mode free decay test results.

Fig. 7  Third mode free decay test results.

Fig. 8  Continuous excitation test results. Vertical dashed lines indicate moment at which controller is turned off.
where the modal vector $q \in \mathbb{R}^{10}$ and modal stiffness matrix $\mathbf{K} \in \mathbb{R}^{10 \times 10}$ were determined through an ordinary modal decomposition performed on the discrete FEM. Equation (21) is merely a discretization of Eq. (12). The modal force vector is defined as $\mathbf{f}_q = [a_1k_1, \ldots, a_{10}k_{10}]$, where $k_j = (x_j)^2$ is formed from experimentally observed open-loop natural frequency values. Actual experimentally obtained open-loop damping coefficients were likewise used to form $\mathbf{C}$. The disturbance force introduced into the system via the drive coil was not included in the model because its inclusion has no bearing on the closed-loop damping coefficients and natural frequencies. Because the measured voltage $V_s(t)$ is related to the current flow through the sensor $i_s$, such that

$$i_s = \frac{1}{C_s} dV_s \frac{dt}{d}$$

where $C_s$ is the NSMS capacitance, then upon discretization of the output equation in Theorem 2, the NSMS measurement is approximated as

$$V_s = -\left(\frac{1}{C_s}\right) \mathbf{f}_q^T q$$

The controller was similarly expressed in the first-order form

$$\dot{x}_c = A_c x_c + B_c V_s$$

where the definitions are obvious. Closed-loop system poles were then found by combining the four preceding equations to form a closed-loop system. Damping factors and natural frequencies corresponding to the (predicted) closed-loop poles are tabulated in Table 3. By introducing a multiplicative factor $k$ into the controller output equation such that $V_s = k C_s x_s$, the root locus of the predicted closed-loop pole locations as a function of $k$ was determined and is given in Fig. 10. Predicted pole locations ($k = 1$) are indicated by a (+) sign, and experimental pole locations are designated with an asterisk (*). The system is unstable for high feedback gains; the predicted gain margin of the experimental controller was found to be 30.447 (29.671 dB).
E. Analysis of Results

As shown in Table 3, the predicted damping coefficient values are close to the experimentally obtained values. The fidelity of the closed-loop eigenvalue predictions renders as indistinguishable the theoretical and measured closed-loop pole locations in Fig. 10. A quantitative comparison between predicted and measured damped natural frequency values yields a worst case 3.28% error for mode 2. Modes 1 and 4 yield error margins less than 0.5%. Several error sources exist, which are likely to account for the discrepancies between the predicted and actual behavior of the experimental system, and are now briefly discussed.

Finite Element Model

FEM mode shape predictions enter directly into the theoretical model via Eq. (18) and hence directly affect the accuracy of the theoretical solution. FEM predicted open-loop natural frequencies of the first four modes are known to deviate as much as 8% from actual measured values. FEM mode shape predictions, and thus MPF predictions, are then reasonably assumed to be similarly accurate to within 8% of actual values.

Numerical Errors

To solve Eq. (18), actual mode shape data obtained through the FEM were fit to a polynomial of the form given in Eq. (20) through a (nonweighted) least-squares minimization. Although the polyonomic functions were constrained to satisfy all geometric boundary conditions, natural boundary conditions were not necessarily satisfied. Hence the polyonomic mode shape approximations are expected to lack some fidelity in predicting the true MPF values associated with the experimental structure, although the resulting margin of error is difficult to quantify.

Data Measurement

The accuracy of measured data is dependent on the fidelity of the discrete data sampling and signal processing methods employed as well as the quality of isolation of the test bed environment. Air currents, floor vibrations, and electromagnetic radiation were seen to induce measurable signals. Although great care was taken to minimize extraneous noise sources and to enhance accurate data collection, some error is inevitably introduced.

In consideration of all of the aforementioned error sources, error margins as large as 10% could have been reasonably assumed. Hence the fidelity of the preceding results falls well within the expected margins and serves to validate the SMT concept as applied to an orthotropic plate.

F. Design Considerations

The final experimental structure design was the outcome of a lengthy evolutionary process. Several findings of that process are now discussed.

Air Gaps

Extreme care must be taken to ensure that no significant air gaps exist between active surfaces, as voltage transients may otherwise develop that will generate enough heat to destroy the integrity of the film. In this regard, the weakest part of any design attempted was found to be the attachment of the lead wire to the surface of the PVDF film. The attachment was made by soldering the lead wire to a thin copper tab and then bonding the tab to the film surface via a silver ink paste. Care must be taken to remove any air gaps that may exist underneath the lead attachment. In an earlier design each layer was folded in half, and lead attachments were buried in the structure in accord with an isotropic beam experimental design used by Lee et al.14 Folding each layer in half and grounding the outer surface provide good shielding against electromagnetic noise, which otherwise would be detected by the sensors. However, burying the lead attachments created air gaps that destroyed film layer integrity. In the present design, lead attachments were exposed so that failed attachments could be easily repaired.

In the present design, the film attachment regions were folded in half to attach lead wires to those film surfaces facing toward the substrate. To insulate the positive surfaces from the electrically grounded substrate, thin pieces of Mylar® were placed underneath the film tabs, which inadvertently created an air gap between the PVDF film and the substrate. Consequently, during the test, a high-voltage transient developed and destroyed the integrity of the film in a local region near the lead attachment point to layer 2. In future experiments it will be advisable to place a Mylar sheet (or other insulatory material) along the entire surface between the substrate and adjoining film layers.

Film Boundaries

Voltage arcing commonly occurs across film edges if care is not taken to remove the (manufacturer-deposited) silver ink electrode coating on the film surfaces near all boundaries. MEK is an excellent solvent for this purpose.

Radiative Noise

The PVDF film layers are highly sensitive to sources of electromagnetic noise. Care should be taken to properly shield the film surfaces. The current design was seen to perform fairly well in this respect.

Lamination

The bonds between laminae must be both thin and uniform (in particular, free of air bubbles). An ordinary slow dry (24-h) epoxy worked very well in this regard. Heat should be applied when mixing the epoxy and hardening agent. The bleeding process (squeezing out excess epoxy) is critical to ensure thin interlaminar regions: weight should be applied evenly to the surface during cure. In the current design, a stress of 0.6 kg/cm² was applied during the dry. Although the bleeding process was excellent in the current design (interlaminar thicknesses on the order of 2–3 μm), the previous Lee et al.14-type folded-layer design was deficient in this respect. In that design, epoxy could not flow freely along the folded edge, and hence interlaminar thicknesses exceeded 20 μm. It was also observed that the epoxy-filled interlaminar regions never were seen to provide any electrical insulation between adjoining layers.

In designing the NSMS, an extremely thin (≤5 μm) double-sided tape was tried as an interlaminar adhesive and was seen to perform well. The tape also provided electrical insulation between layers. Bonding appeared to be excellent. Nonetheless, this approach cannot be used for high voltage applications and is assumedly inappropriate when specimen longevity is a primary factor.

VI. Conclusions

A general SMT design methodology recently introduced by the authors for piezolaminated anisotropic plate systems was reviewed and then extended to provide the means to predict the modal characteristics of multilayered piezolaminated transducers embedded in an anisotropic plate. The SMT theory was then validated through experimental test. An experimental procedure was described involving a 10-layered orthotropic plate constructed from 4 graphite-epoxy layers sandwiched between 6 piezoelectrically active PVDF sublaminae. Three PVDF sublaminae stacked on each face were combined electrically to provide both a sensed measurement and an actuation channel with predicted modal content. Lead compensation was employed to provide active control. Open- and closed-loop frequency and transient response data were collected, and significant active vibration attenuation was observed. A numerical simulation directly based on an SMT-derived transducer model was developed and then compared with the actual system behavior. The theoretically based numerical results were seen to closely resemble the measured response and to fall within expected error bounds. Several implementation issues were identified and discussed.

References


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